The Birch and Swinnerton-Dyer Conjecture

by

A. Wiles

A polynomial relation $f(x, y) = 0$ in two variables defines a curve C_0. If the coefficients of the polynomial are rational numbers then one can ask for solutions of the equation $f(x, y) = 0$ with $x, y \in \mathbb{Q}$, in other words for rational points on the curve. The set of all such points is denoted $C_0(\mathbb{Q})$. If we consider a non-singular projective model C of the curve then topologically C is classified by its genus, and we call this the genus of C_0 also. Note that $C_0(\mathbb{Q})$ and $C(\mathbb{Q})$ are either both finite or both infinite. Mordell conjectured, and in 1983 Faltings proved, the following deep result

Theorem [F1]. *If the genus of C_0 is greater than or equal to two, then $C_0(\mathbb{Q})$ is finite.*

As yet the proof is not effective so that one does not possess an algorithm for finding the rational points. (There is an effective bound on the number of solutions but that does not help much with finding them.) The case of genus zero curves is much easier and was treated in detail by Hilbert and Hurwitz [HH]. They explicitly reduce to the cases of linear and quadratic equations. The former case is easy and the latter is resolved by the criterion of Legendre. In particular for a non-singular projective model C we find that $C(\mathbb{Q})$ is non-empty if and only if C has p-adic points for all primes p, and this in turn is determined by a finite number of congruences. If $C(\mathbb{Q})$ is non-empty then C is parametrized by rational functions and there are infinitely many rational points. The most elusive case is that of genus 1. There may or may not be rational solutions and no method is known for determining which is the case for any given curve. Moreover when there are rational solutions there may or may not be infinitely many. If a non-singular projective model C has a rational point then $C(\mathbb{Q})$ has a natural structure as an abelian group with this point as the identity element. In this case we call C an elliptic curve over $\mathbb{Q}$. (For a history of the development of this idea see [S]). In 1922 Mordell ([M]) proved that this group is finitely generated, thus fulfilling an implicit assumption of Poincaré.

Theorem. *If C is an elliptic curve over $\mathbb{Q}$ then*

$$C(\mathbb{Q}) \simeq \mathbb{Z}^r \oplus C(\mathbb{Q})^{\text{tors}}$$

for some integer $r \geq 0$, where $C(\mathbb{Q})^{\text{tors}}$ is a finite abelian group.

The integer r is called the rank of C. It is zero if and only if $C(\mathbb{Q})$ is finite. We can find an affine model for an elliptic curve over $\mathbb{Q}$ in Weierstrass form

$$C\colon y^2 = x^3 + ax + b$$

with $a, b \in \mathbb{Z}$. We let Δ denote the discriminant of the cubic and set

$$\begin{aligned} N_p &:= \#\{\text{solutions of } y^2 \equiv x^3 + ax + b \bmod p\} \\ a_p &:= p - N_p. \end{aligned}$$

Then we can define the incomplete L-series of C (incomplete because we omit the Euler factors for primes $p|2\Delta$) by

$$L(C, s) := \prod_{p \nmid 2\Delta} (1 - a_p p^{-s} + p^{1-2s})^{-1}.$$

We view this as a function of the complex variable s and this Euler product is then known to converge for $\mathrm{Re}(s) > 3/2$. A conjecture going back to Hasse (see the commentary on 1952(d) in [We1]) predicted that $L(C, s)$ should have a holomorphic continuation as a function of s to the whole complex plane. This has now been proved ([W], [TW], [BCDT]). We can now state the millenium prize problem:

Conjecture (Birch and Swinnerton-Dyer)**.** *The Taylor expansion of $L(C, s)$ at $s = 1$ has the form*

$$L(C, s) = c(s-1)^r + \text{ higher order terms}$$

with $c \neq 0$ and $r = \mathrm{rank}(C(\mathbb{Q}))$.

In particular this conjecture asserts that $L(C, 1) = 0 \Leftrightarrow C(\mathbb{Q})$ is infinite.

Remarks. 1. There is a refined version of this conjecture. In this version one has to define Euler factors at primes $p|2\Delta$ to obtain the completed L-series, $L^*(C,s)$. The conjecture then predicts that $L^*(C,s) \sim c^*(s-1)^r$ with

$$c^* = |Ш_C| R_\infty w_\infty \prod_{p|2\Delta} w_p / |C(\mathbb{Q})^{\mathrm{tors}}|^2.$$

Here $|Ш_C|$ is the order of the Tate-Shafarevich group of the elliptic curve C, a group which is not known in general to be finite although it is conjectured to be so. It counts the number of equivalence classes of homogeneous spaces of C which have points in all local fields. The term R_∞ is an $r \times r$ determinant whose matrix entries are given by a height pairing applied to a system of generators of $C(\mathbb{Q})/C(\mathbb{Q})^{\mathrm{tors}}$. The w_p's are elementary local factors and w_∞ is a simple multiple of the real period of C. For a precise definition of these factors see [T1] or [T3]. It is hoped that a proof of the conjecture would also yield a proof of the finiteness of $Ш_C$. 2. The conjecture can also be stated over any number field as well as for abelian varieties, see [T1]. Since the original conjecture was stated much more elaborate conjectures concerning special values of L-functions have appeared, due to Tate, Lichtenbaum, Deligne, Bloch, Beilinson and others, see [T2], [Bl] and [Be]. In particular these relate the ranks of groups of algebraic cycles to the order of vanishing (or the order of poles) of suitable L-functions. 3. There is an analogous conjecture for elliptic curves over function fields. It has been proved in this case by M. Artin and J. Tate [T1] that the L-series has a zero of order at least r, but the conjecture itself remains unproved. In the function field case it is now known to be equivalent to the finiteness of the Tate-Shafarevich group, [T1], [Mi] III corollary 9.7. 4. A proof of the conjecture in the stronger form would give an effective means of finding generators for the group of rational points. Actually one only needs the integrality of the term $Ш_C$ in the expression for $L^*(C,s)$ above, without any interpretation as the order of the Tate-Shafarevich group. This was shown by Manin [Ma] subject to the condition that the elliptic curves were modular, a property which is now known for all elliptic curves by [W], [TW], [BCDT]. (A modular elliptic curve is one which occurs as a factor of the Jacobian of a modular curve.)

Early History

Problems on curves of genus 1 feature prominently in Diophantus' Arithmetica. It is easy to see that a straight line meets an elliptic curve in three points (counting multiplicity) so that if two of the points are rational then so is the third.[1] In particular if a tangent is taken to a rational point then it meets the curve again in a rational point. Diophantus implicitly uses this method to obtain a second solution from a first. However he does not iterate this process and it is Fermat who first realizes that one can sometimes obtain infinitely many solutions in this way. Fermat also introduced a method of 'descent' which sometimes permits one to show that the number of solutions is finite or even zero. One very old problem concerned with rational points on elliptic curves is the congruent number problem. One way of stating it is to ask which rational integers can occur as the areas of right-angled triangles with rational length sides. Such integers are called congruent numbers. For example, Fibonacci was challenged in the court of Frederic II with the problem for $n = 5$ and he succeeded in finding such a triangle. He claimed moreover that there was no such triangle for $n = 1$ but the proof was fallacious and the first correct proof was given by Fermat. The problem dates back to Arab manuscripts of the 10^{th} century (for the history see [We2] chapter 1, §VII and [Di] chapter XVI). It is closely related to the problem of determining the rational points on the curve C_n: $y^2 = x^3 - n^2 x$. Indeed

$$C_n(\mathbb{Q}) \text{ is infinite} \Longleftrightarrow n \text{ is a congruent number}$$

Assuming the Birch and Swinnerton-Dyer conjecture (or even the weaker statement that $C_n(\mathbb{Q})$ is infinite $\Leftrightarrow L(C_n, 1) = 0$) one can show that any $n \equiv 5, 6, 7 \bmod 8$ is a congruent number and moreover Tunnell has shown, again assuming the conjecture, that for n odd and square-free

$$n \text{ is a congruent number} \Longleftrightarrow \begin{array}{l} \#\{x,y,z \in \mathbb{Z}\colon 2x^2 + y^2 + 8z^2 = n\} \\ \qquad = 2 \times \#\{x,y,z \in \mathbb{Z}\colon 2x^2 + y^2 + 32z^2 = n\}, \end{array}$$

with a similar criterion if n is even ([Tu]). Tunnell proved the implication left to right unconditionally with the help of the main theorem of [CW] described below.

Recent History It was the 1901 paper of Poincaré [P] which started the modern interest in the theory of rational points on curves and which first raised questions about the minimal

[1] This was apparently first explicitly pointed out by Newton.

number of generators of $C(\mathbb{Q})$. The conjecture itself was first stated in the form we have given in the early 1960's (see [BS]). In the intervening years the theory of L-functions of elliptic curves (and other varieties) had been developed by a number of authors but the conjecture was the first link between the L-function and the structure of $C(\mathbb{Q})$. It was found experimentally using one of the early computers EDSAC at Cambridge. The first general result proved was for elliptic curves with complex multiplication. (The curves with complex multiplication fall into a finite number of families including $\{y^2 = x^3 - Dx\}$ and $\{y^2 = x^3 - k\}$ for varying $D, k \neq 0$.) This theorem was proved in 1976 and is due to Coates and Wiles [CW]. It states that if C is a curve with complex multiplication and $L(C, 1) \neq 0$ then $C(\mathbb{Q})$ is finite. In 1983 Gross and Zagier showed that if C is a modular elliptic curve and $L(C, 1) = 0$ but $L'(C, 1) \neq 0$, then an earlier construction of Heegner actually gives a rational point of infinite order. Using new ideas together with this result, Kolyvagin showed in 1990 that for modular elliptic curves, if $L(C, 1) \neq 0$ then $r = 0$ and if $L(C, 1) = 0$ but $L'(C, 1) \neq 0$ then $r = 1$. In the former case Kolyvagin needed an analytic hypothesis which was confirmed soon afterwards; see [Da] for the history of this and for further references. Finally as noted in remark 4 above it is now known that all elliptic curves over $\mathbb{Q}$ are modular so that we now have the following result:

Theorem. *If $L(C, s) \sim c(s - 1)^m$ with $c \neq 0$ and $m = 0$ or 1 then the conjecture holds.*

In the cases where $m = 0$ or 1 some more precise results on c (which of course depends on the curve) are known by work of Rubin and Kolyvagin.

Rational Points on Higher Dimensional Varieties We began by discussing the diophantine properties of curves, and we have seen that the problem of giving a criterion for whether $C(\mathbb{Q})$ is finite or not is an issue only for curves of genus 1. Moreover according to the conjecture above, in the case of genus 1, $C(\mathbb{Q})$ is finite if and only if $L(C, 1) \neq 0$. In higher dimensions if V is an algebraic variety, it is conjectured (see [L]) that if we remove from V (the closure of) all subvarieties which are images of $\mathbb{P}^1$ or of abelian varieties then the remaining open variety W should have the property that $W(\mathbb{Q})$ is finite. This has been proved in the case where V is itself a subvariety of an abelian variety by Faltings

[F2]. This suggests that to find infinitely many points on V one should look for rational curves or abelian varieties in V. In the latter case we can hope to use methods related to the Birch and Swinnerton-Dyer conjecture to find rational points on the abelian variety. As an example of this consider the conjecture of Euler from 1769 that $x^4 + y^4 + z^4 = t^4$ has no non-trivial solutions. By finding a curve of genus 1 on the surface and a point of infinite order on this curve, Elkies [E] found the solution,

$$2682440^4 + 15365639^4 + 18796760^4 = 20615673^4$$

His argument shows that there are infinitely many solutions to Euler's equation. In conclusion, although there has been some success in the last fifty years in limiting the number of rational points on varieties, there are still almost no methods for finding such points. It is to be hoped that a proof of the Birch and Swinnerton-Dyer conjecture will give some insight concerning this general problem.

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Lesson1

These lessons are based on Vedic Maths" principles and other maths tricks.These principles are general in nature and can be applied in many ways and very very useful in commercial arthematics**.** I hope all of you like these lessons and make your calculation more fast and save lot of time in daily calculations and examinations or any entrance test like **CAT /IIT /BANK PO /ENGINEERING ENTRANCE TEST/PMT /MCA ENTRANCE TEST/MBA ENTRANCE TEST etc etc**

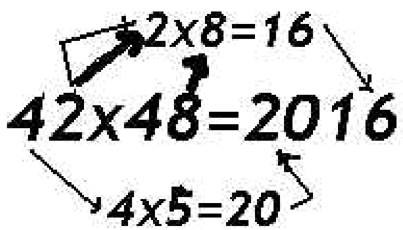
Method for multiplying numbers where the first figures are same and the last figures add to1 0

. **42 x 48 =**

Both numbers here start with 4 and the last figures (2 and 8) add up to 10.

just multiply 4 by 5 (the next number up) to get **20** for the first part of the answer.

And we multiply the last figures: 2 x 8 = **16** to get the last part of the answer



Method for multiplying numbers where the first figures add up 10 and the last figures are same

44X64

Here first figures are 4 and 6 and their add up 10 and unit figures of both number are same Just multiplying the last figures 4x4=16 Put it at right hand side

Again multiplying the first figures and add common degit(4x6)+4=24+4=28 put it at left hand side

Now we get required answer2816

Similarly 36x76 , 6X6 =36 right hand side , (3x7)+6= 21+6=27 left hand side

Required answer is 2736

NOTE If multiplication of last figures is less than 10 add zero before unit digit

Ex 81x21 , 1x1=01,(8x2)+1= 16+1=17 Required answer 1701

Method for multiplying numbers where the first number"s add up10 and and the second number's digits are same

46X55

Here first number's add up is 10 and second number "s digits are common i.e 5

Just multiplying last figures of both numbers 6x5 =30 put it at right hand side

Again multiplying first figures of both numbers and add common digit of second number

(4x5)+5 =20+5 =25 put it left hand side

Required answer is 2530 (If multiplication is in unit in first step add zero before it)

Multiplying numbers just over 100.

108 x 109 = 11772

The answer is in two parts: 117 and 72,
117 is just 108 + 9 (or 109 + 8),
and 72 is just 8 x 9.

Similarly **107 x 106 = 11342**

HOW USEFUL IS THIS CALCULATION!
EXAMPLE 1

Compute the amount and the compound intrest on Rs 10000.00 on 2.years at4%per annum.

$$=10000\times\left(1+\frac{4}{100}\right)^2$$

$$=\frac{1\cancel{0000}\times104\times104}{1\cancel{00}\times1\cancel{00}}$$

$$=104\times104$$

Now 4x4=16 and 4+4=8 put 10 it becomes Rs 10816.00
How simple ! no calculation no extra time !

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Practice Test 1
Solve mentally these questions
Note down your calculation time by watch
(Suggested time 90 seconds)

22 X 28	**35 X 35**	**48 X 42**
73 X 33	**12 X 92**	**48 X 28**
28 X 22	**37 X 88**	**91 X 66**
118 X105	**109 X108**	**112 X106**
91 X 99	**33 X 37**	**55X55**

Now check your answer with the help of a calculator

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Lesson2

Multiplying a number by 11

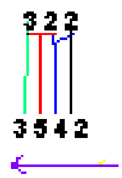
To multiply any number by 11 we just put
the total of the two figures between the 2 figures.

32X11

Here first we write 2 of 32 extreme right ,now add up 3+2 =5 ,put 5 before 2 and write 3 extreme left .Required answer is 352

322x11

Write extreme right 2of 322 and add up 2&2 of 322 , 2+2=4 ,write 4 before 2 Now our partial answer is ....42 .Again add up 3&2 of 322 .3+2 =5 write before ..42 .Now our partial answer becomes ..542 .Put 3 before 5 . Now our required answer is 3542



- **77 x 11 = 847**

This involves a carry figure because 7 + 7 = 14
we get 77 x 11 = 7_147 = 847

Multiplication of 22 ,33 ,44 ,55, 66 , 77 ,88

I think all of you understand this concept . It is a very useful tool ,with the help of this tool now we learn multiplication of 22 ,33 .44 ,55. 66 . 77 ,88

22x45 =(2x11)x45 = 11x(2x45)=11x90= 990
33x66 = (3x11)x66 = 11x(3x66)= 11x198= 2178
44x356=(4x11)x356= 11x(4x356)=11x 14 24=1 5664
55x58 =(5x11)x28 = 11x(5x28) = 11x 140 = 1540
66x23 =(6x11)x23 = 11x(6x23) = 11x138 = 1518
Same way 77 =7x11 and 88=8x11

Multiplication of 11 is great time saver How ???

You can do calculations in Mensuration very fast by this tool
As you know value of π is 22/7 .You can break it as 2x11/7
Ex Find the area of circle of radius of 14 cm

(22/7)x14x14=11x2x2x14=11x56=616square cm
All calculation mentally, no rough work , a great time saver.

Practice Test 2

Solve mentally these questions

Note down calculation time by your watch

(Suggested time 180 seconds)

11 X 356	**11 X 9587**	**587 X 22**
33 X 52	**44 X 897**	**758 X 88**
121 X789*	**11 X 9874**	**487 x99**

Find the volume of a sphere of radius of 10.5 cm*

*Hints 121 X 789 = 11 X11 X789 = 11 X 8679=95469

volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4 \times 10.5 \times 10.5 \times 10.5 \times 22}{3 \qquad\qquad 7}$

= 4 x0.5 x 10.5x 10.5 x 22

= 2x2 x0.5 x 10.5x10.5 x22

= (2x 10.5)(2x10.5)(0.5x22)

= 21 x21 x11

= 441 x11

=4851 cube cm

It seems to quite lengthy , actually it takes very little time . It is very tough to calculate 10.5x10.5 x10.5 conventionally.

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Lesson3

Multiplication by 5

It's often more convenient instead of multiplying by 5 to multiply first by 10 and then divide by 2. For example, 237x5=2370/2=1185
Same way you can also multiply by 50

Multiplication by 4

Replace either with a repeated operation by 2.For example 124x4=248·2=4

Multiplication by 25

Multiply first by 100 and then divide by 4 For example37x25=3700/4=1850/2=925.

Multiplication by 8

Replace either with a repeated operation by 2. For example 124x8=248x4=496x2=992.

Multiplication by 45

Multiply first by 90and then divide by 2.For example37x45=37x90/2=3330/2=1665
Or see this method also 37 x45 = 37 (100/2 -10/2)= 3700/2-370/2=1850-185=1665

Multiplication by 55

Multiply first by 110and then divide by 2. For example37x55=37x110/2=4070/2=2035

Multiplication by 75

Multiply first by 300 and then divide by 4 .For example37x75=37x300/4=11100/4=2775

Multiplication by 125

Multiply first by 500 and then divide by 4 For example37x125=37x500/4=18500/4=4625

Multiplication by 150

Multiply first by 300 and then divide by 2 For example37x150=37x300/4=11100/2=5550

Multiplication by 175

Multiply first by 700 and then divide by 4 For example37x175=37x700/4=25900/4=6475

Multiplication by 225

Multiply first by 900 and then divide by 4 For example37x225=37x900/4=33300/4=8325

Multiplication by 275

Multiply first by 1100 and then divide by 4
For example37x275=37x1100/4=40700/4=10175

A Great Time Saver

Q A music shop has a sound system marked at Rs. 14000 on which a sales tax of 12.5% is chargeable. Find the total value a customer would have to pay for the sound system.

$$\text{sales tax} = \frac{14000\text{x}12.5}{100} = \frac{14000\text{x}125}{1000} = \frac{14\text{x } 500}{4} = \frac{7000}{4} = 1750$$

Rs 14000+1750 =Rs15750 Answer

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Lesson4

MULTIPLICATION OF 12 TO 19

MULTIPLICATION BY 12:

The method is exactly the same as in the case of 11 except that you double each number before adding the right neighbour.
Step1 Prifix 0 before multiplicand

step2: Multiply last digit by 2 and write it at extreme right take carry if there is
Step3: Double and add the right neighbour with carry in step first repeat this upto the first digit ie till 0

. **1234 x12=** 01234x12 =

Here 2 multiply by 4 write 8 at extreme right 8
Double 3 and add to 4 and write before...8,6+4=10 here we write 0 before 8 and take 1 carry 08
Double 2 and add to 3 and add carry1 and write before 08 ,4 +3 +1 =8 808
Double 1and add to 2 and write before ...708, 2+ 2 =4 4808
Double 0 and add to 1 and write before .708 0+1=1 14808

14808 is our required answer

MULTIPLICATION FROM 13 TO 19:

The method is exactly the same as in the case of 12 except that you treble each degit before add in the right neighbour, in the case of multiplication of 13 .We would quadruple (i.e. multiply by 4) and then add the right neighbour, in the case of multiplication of 14 .In case of 19 we would multiply by 9 and then add the right neighbour

39942 X 13 = ? 039942x13=

2x3 =6 6
4x3+2 =14 46
9x3+4+1 =32 246
9x3+9+3=39 9246
3x3+9+3=21 19246
0x3+3+2=5 519246

519246 is our required answer.

34x17 =? 034x17

4x7 =28 8
3x7+4+2 =27 78
0x7 +3+2=5 578

578 is our required answer.

Now try these examples

1) 39942 X 13 = ? (2) 43285 X 14 = ?

2331	21132
039942 X 13	043285 X 14
519246	605990

(3) 58265 X 15 = ?
34132
<u>058265</u> X 15
<u>873975</u>

(4) 36987 X 16 = ?
25654
<u>036987</u> X 16
<u>591792</u>

(5) 69873 X 17 = ?
57652
<u>069873</u> X 17
<u>1187841</u>

(6) 96325 X 18 = ?
85224
<u>096325</u> X 18
<u>1733850</u>

(7) 74125 X 19 = ?
73124
<u>074125</u> X 19

Practice Test

24678 x 13=	246x13x 15=	89x19 x18=
98745 x14=	987 x19 =	266x16=
98745 x17=	8745x 12 =	3435 x12=

Check your answer with the help of calculator

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Lesson5

MULTIPLICATION OF 9

Step1 Prifix 0 before multiplicand
Step2 Take last digit, subtract it from 10 and write it at the extreme right
Step3 Take second last digit add one and subtract it from last digit write before step 1 figure
Step4 Now move toward left take third last digit and subtract it from right neighbour ,write before step 2 figure and repeat it up to first digit ie 0

123456 x9 = ??
0123456x9 =

Here subtract 6 from 10 (10-6=4) write 4 at the extreme right			4
Take 5 add 1 (5+1 =6) and subtract it to last digit 6 and write before ..4,6-6=0			04
Take 4 subtract it from 5 and write before 04		5-4 =1	104
Subtract 3 from 4	and write before ...104	4-3 =1	1104
Subtract 2 from 3	and write before ..1104	3-2 =1	11104
Subtract 1 from 2	and write before..11104	2-1 =1	111104
Subtract 0 from 1	and write before.111104	1-0 =1	1111104

1111104 is our required answer

365 x9= ??
0365x9=

10-5 =5		5
6+1=7 subtract 7 from 5 , take carry from 6 now 5 become 15 (15- 7 =8)		85
Now 6 become 5(adjust carry) subtract 3 from it	5-3 =2	285
3-0 =3		3285

3285 is our required answer

I think all of you understand this concept . It is a very useful tool ,with the help of this tool now we learn multiplication of 36 ,.45 ,54. 63 . 72 ,81 and other multiple of 9
212x36 =(4x9)x212 = 9x(848)=7632
212x45 = (90/2)x212 = 9x(2120/2)= 9x1060= 9540
43x54=(9x6)x43= 9x(43x6)=9x 258=2322
63x58 =(9x7)x58 = 9x(7x58) = 9x 406 = 3654
72x23 =(8x9)x23 = 9x(8x23) = 9x184 = 1656
81 x59 =(10-1)9 x59 = (590-59)x9=531x9 = 4779

Practice Test

2589 x9=	5897 x36 =	569x9=
256x45 =	987x63 =	177 x9=
458x54 =	458x9=	2597x72=

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Lesson6

Multiplying Close Numbers

Here we learn how to calculate close numbers multiplication like 96x92, 43x49 ,88x98

Multiply 88 by 98.

Both 88 and 98 are close to 100.
88 is 12 below 100 and 98 is 2 below 100.

88 - 12
98 - 2
86 24

As before the **86** comes from
(or 98 - 12 = 86: you can subtract
either way, you will always get
the same answer).
And the **24** in the answer is
just 12 x 2: you multiply vertically.
So **88 x 98 =** **<u>8624</u>**

93 x96 =??

9 3 + 7

9 6 + 4

~~89 28~~ our required answer is 8928 .Here our base is 100. Now we consider some other base

Ex:- 53 / + 3
57 / + 7
60 / +21
× 50
3000 / +21

Here base is shifted to 50 .so we multiply 60 by 50 and add 21 60x30 +21
Therefore, the answer is **3021**.

See one more example

54 / +4

48 / -2
52 / -8
x50
2600 /-8 2600-8 =2592 Therefore the answer is 2592

Again see one more example
48 /-2

45 /-5
43 /+10
x 50 Base is 50
2150 +10 = 2160

We can do this example taking base 40
48 / +8
45/ + 5
53 / +40
x40
2120 +40 =2160 Why we take base 50 instead of 40 ? Reason is simple calculation of 50 is easy compare to 40.

Let us consider another example for practice.

Ex:- 82 / + 2 Shifted base = 80
76 / - 4
78 / (-8)
× 80
6240 / (-8) ie 6240
- 8
Answer is **6232**

Advantages : -

(1) The computation time is reduced drastically.
(2) This method is particularly useful when the numbers under consideration are close to base.
(3) Very useful in finding the squares.

Disadvantage : -

The disadvantage of this method is that the numbers under consideration should be very close to each other. If there is a large duration in the numbers from each other then it is very difficult to fix the base for ex:- 32 × 78 etc.

Practice Test

96x 98	97 x 88	91X93	96 x 93	89x92
52 X49	73x82	45x48	69x72	78x79
45x42	72x75	48 x52	95x88	82x75

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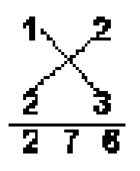
Lesson7

VERTICALLY AND CROSSWISE MULTIPLICATION

Consider the conventional multiplication of two 2 digit numbers 12 and 23 shown below:

```
  12
X 23
  36
 24x
 276
```

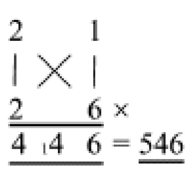
This is normally called long multiplication but actually the answer can be written straight down using the VERTICALLY AND CROSSWISE formula.



There are 3 steps:

a) Multiply **vertically on the left**: 2 x 3= **6**
This gives the first figure of the answer.
b) Multiply **crosswise and add**: 1 x 3 + 2 x 2 = **7**
This gives the middle figure.
c) Multiply **vertically on the right**: 1 x 2 = **3**
This gives the last figure of the

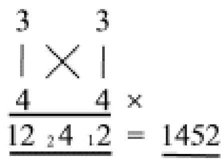
- **21 x 26 = 546**



The method is the same as above
except that we get a 2-figure number, 14, in the middle step, so the 1 is carried over to the left (4 becomes 5).

- **33 x 44 = 1452**

  There may be more than one carry in a sum:



Vertically on the left we get 12.
Crosswise gives us 24, so we carry 2 to the left
and mentally get 144.

Then vertically on the right we get 12 and the 1
here is carried over to the 144 to make 1452.

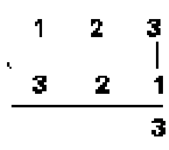
This is multiplication of 2 digits numbers .Now we learn 3 digits multiplication

3 Digits Multiplication

Let us try to understand this by an example.

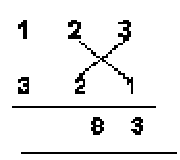
123 x321=????

Step 1 We start from right and multiply last digits of both numbers vertically



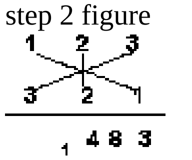
3x1 =3

Step 2 Multiply last two digits crosswise and add together and write before step one figure



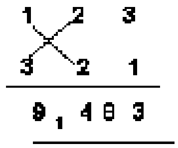
2x1+3x2=2 + 6 = 8

Step 3 Multiply first and last digits crosswise and middle digit vertically and add together and write total before step 2 figure



1x1 +3x3 +2x2 =1+9+3=14

Step 4 Multiply first two digits crosswise and add together and write before step 3 figure also add carry



2x1 +3x2+1 =9

Step 5 Multiply first digits of both numbers vertically and write before step 4 figure

1 2 3
I
3 2 1
3 9 1**4 8 3** 1x3=3

Our required answer is 39483

Practice Test

28 x89 28x62 56x48 236 x456 58x 78

259x 789 289 x89 * 87 x56 211 x879 25x89

Hint 289 x089
289
x 089

0x2 / 2x8+8x0=16 / 2x9+8x8+0x9=82 / 8x9+8x9=144 / 9x9=81

16+9=**25** / 82+15=9**7** /144+8 =15**2** /8**1**

= **25721**

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Lesson8

Let us try a long multiplication 235465 x233297 =????
This is the conventional method and everybody know this method so I am skipping the explanation.

```
       2 3 5 4 6 5
     x_2_3_3_2_9_7_
     1 6 4 8 2 5 5
    21 1 9 1 8 5 x
    47 0 9 3 0 x x
   706 3 9 5 x x x
  7063 9 5 x x x x
_47093_0_x_x_x_x-_x_
_54933_2_7_8_1_0_5_
```

Now the real magic of vedic maths come here and you can solve this problem in few seconds

Steps Almost same as we do in previous lesson .Start from right ,multiply 5 to7 (5x7=35) write 5 take carry3

```
  2  3  5  4  6  5
x_2_3_3_2_9_7
                35
```

Now multiply 6 to 7 ,9 to 5 add together with carry 3
6x7+9x5 +3 =42 +45+3=90 write 90 before 5

```
  2  3  5  4  6  5
x_2_3_3_2_9_ 7
              9035
```

Now multiply 4to5 ,5 to2 ,6 to 9 add together with carry9
4x7+5x2+6x9+9= 28+10+54+9=101

```
  2  3  5  4  6  5
x_2_3_3_2_9_ 7
        1019035
```

Now 5x7+4x9+6x2+5x3+10 =35 +36+12+15+10=108

```
  2  3  5  4  6  5
x_2_3_3_2_9_ 7
     1081019035
```

Now 3x7+5x9+4x2+6x3+5x3+10=21 +45+8+18+15+10=117

```
  2  3  5  4  6  5
```

x 2 3 3 2 9 7

11**7**10**8**10**1**9**0**3**5**

2x7+3x9+5x2+4x3+6x3 +5x2+11=14+27+10+12+18+10+11=102

2 3 5 4 6 5

x 2 3 3 2 9 7

10**2**11**7**10**8**10**1**9**0**3**5**

2x9+3x2+5x3+4x3+6x2+10=18+6+15+12+12+10 =73

2 3 5 4 6 5

x 2 3 3 2 9 7

7**3**10**2**11**7**10**8**10**1**9**0**3**5**

2x2+3x3+5x3+4x2+7= 4+9+15+8+7=43

2 3 5 4 6 5

x 2 3 3 2 9 7

4**3**7**3**10**2**11**7**10**8**10**1**9**0**3**5**

2x3+3x3+5x2+4=6+9+10+4=29

2 3 5 4 6 5

x 2 3 3 2 9 7

2**9**4**3**7**3**10**2**11**7**10**8**10**1**9**0**3**5**

2x3+3x2+2=6+6+2=14

2 3 5 4 6 5

x 2 3 3 2 9 7

1**4**2**9**4**3**7**3**10**2**11**7**10**8**10**1**9**0**3**5**

2x2+1=5

2 3 5 4 6 5

x 2 3 3 2 9 7

51**4**2**9**4**3**7**3**10**2**11**7**10**8**10**1**9**0**3**5**

Our required answer is 5493327109

It looks quite boring and tedious .But certainly a great time saver, do practice and become master

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Lesson9

TO CALCULATE THE PRODUCT OF NOs. Ending with 5

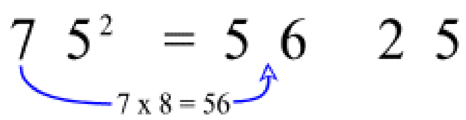
Here we discuss the multiplication of two numbers ending with 5 ie 25x25 ,25x45 35 x75
we cover three types of problems under this heading

1. 25x25 both nos are same no difference ie difference is 0(25-25 =0)
2. 25x35 45x75 difference of first digits (35-25) is odd like **1**0 ,**3**0 ,**7**0,**9**0 ,**11**0
3. 25x45 45 x65 difference of first digits (45-25) is even like **2**0, **6**0 ,**4**0 ,**12**0

Let us take first case when there is no difference

75x75 = 5625

The answer is in two parts: 56 and 25.
The last part is always **25**.
The first part is the first number, 7, multiplied by the number "one more", which is 8:
so 7 x 8 = **56**



Similarly **85x85 = 7225** because 8 x 9 = 72.

Now we take both cases where difference of first digit is either even or odd
When difference of first digit is odd our last part of answer is 75 and in the case of even it is 25
you can remember it by this way - difference of first digits is odd , take 75 having first digit odd
difference of first digist is even, take 25 having first digit even

Now Learn How to calculate first part

See this example 25x45 =?????? 45 -25 =**2**0

Because first digit's difference is even ,our second part of answer is 25
Multiply 2 to 4 ie 2x4 =8, take average of first digit of both numbers (2+4) /2 = 6/2=3
this 3 also add to 8 ie 8+3 =11 It is first part of our answer . **Now our required answer is 1125**

See one more example 45 x85 =??? Difference is even 8-4 =4 second part is 25

8x4 =32 (8+4)/2 =12 /2=6 32 +6 =38

Required answer is 3825

Now see the odd difference example 45 x55=??? 5-4 =1, difference is odd so second part is 75
Multiply 5x4 = 20 now take average of both first digits numbers after deducting 1
(5+4-1)/2 =8/2=4 add this one to multiplication of 5and 4 ie 20 +4 =24
It is our first part Now answer is 2475

Required answer is 2475

See one more example **65 x95 =?????**

Difference is odd 9-6 = 3 so second part is 75
6x9 = 54 , (6+9-1)/2 =(15-1)/2 = 14/2 =7 54 +7 = 61

Required answer is 6175

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Lesson10

Squares

Finding square of any number is very very easy and you can calculate it very fast But before going to concept ,you should do some home work . You are well conversant in squaring of 1 to 9

1x1 =1
2x2 =4
3x3 =9
4x4 =16
5x5 =25
6x6 =36
7x7 =47
8x8 =64
9x9 =81

These are square from 1 to 9 . Now learn square index of 1 to 9

Number	Square Index	Operation
1	2	First digit squaring 1 to4
2	4	
3	6	
4	8	
5	0	Multiply first digit to next to first digit
6	2	
7	4	
8	6	
9	8	5 to9

Square Index is nothing except table of 2 .We just double number in first part ie from 1 to 4 and leave ten 's digit in second part ie from 5 to 9 after doubling .

Steps **Now learn main concept by this example**

21 x21 =????

Here our last digit (or unit digit) is1 .Its square index is 2 First we take square of 1 and write it extreme right then we take first digit and multiply it by the square index write before last figure then squaring first digit and write extreme left If there is any carry add to it

1x1 =**1** 2x2 =**4** 2^2 =**4** **4 4 1 Answer**

44 X44=????
Here square index of 4 is 8

4x4 = 16 4x8 +1(carry) =32 +1 = 33 $4^2 + 3$ =16+3 =**19** **1936** Answer

93 X93 =????
Here square index of 3 is 6

3x3 =**9** 9x6 =5**4** 9^2+ 5 =81 +5 =**86** **8649 Answer**

These are the square where last digit is 1 to 4 now we learn squaring of those numbers having last digit from 5 to 9.Method is same, instead of squaring first digit multiply to next digit

36 X36 =

Here square index of 6 is 2

6x6 =3**6** 3x2 + 3 =6+3=**9** 6X(6+1) = 6 x7 =**42** **42 96 answer**

87 x 87 =

Here square index is 4

7x7=4**9** 8x4+4 =32+4 =3**6** 8x(8+1)+3 =72+3 =**75** **75 69 Answer**

25x25 =

Here square index is 0 so no need of middle step

5x5 = **25** 2x(2+1) = **6** **625 answer**

Now with the help of this concept we can calculate square of any number
236 x 236=????

Here square index of 6is 2
6x6 = 3**6** 23 x2 +3 =46+3=4**9** 23x24*+4 =552+4=**556** **55696** **answer**

*23x24 = ? By cross and vertical multiplication method(covered in lesson no 7)
4 / 14 /12 = 552

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Lesson11

Can it Divide?

How can you *quickly* know if one number will divide evenly into another number, leaving no remainder? For example, will 3 divide evenly into 2,169,252? Well, I wouldn't have brought the subject up if I didn't know some curious shortcuts.I think all of you know the division by 2 ,5 ,10 . Now we learn division by every possible number

Division by 2
No surprise here. Any number that ends in 0,2,4,6 or 8 is evenly divisible by 2.

Division by 3
Add the number's digits. If the sum is evenly divisible by 3, then so is the number. So, will 3 divide evenly into 2,169,252? Yes it will, because the sum of the digits is 27, and 27 is divisble by 3. If you want, you can keep adding numbers until one digit remains. For example, keep going with 27. 2 + 7 = 9, which is also evenly divisible by 3.

Division by 4
If the number's last 2 digits are 00 or if they form a 2-digit number evenly divisible by 4, then number itself is divisible by 4. How about 56,789,000,000? Last 2 digits are 00, so it's divisible by 4. Try 786,565,544. Last 2 digits, 44, are divisible by 4 so, yes, the whole number is divisible by 4.

Division by 5
Any number that ends in a 0 or 5 is evenly divisible by 5. Easy enough.

Division by 6
The number has to be even. If it's not, forget it. Otherwise, add up the digits and see if the sum is evenly divisible by 3. It it is, the number is evenly divisible by 6. Try 108,273,288. The digits sum to 39 which divides evenly into 13 by 3, so the number is evenly divisible by 6. If you want, you can keep adding numbers until only one digit remains and do the same thing. So in this case, 3 + 9 = 12 and 1 + 2 = 3, and 3 is evenly divisible by 3!

Division by 7
Multiply the last digit by 2. Subtract this answer from the remaining digits. Is this number evenly divisible by 7? If it is, then your original number is evenly divisible by 7. Try 364. 4, the last digit, multiplied by 2 = 8. 36, the remaining digits, minus 8 = 28. The last time I checked, 28 is evenly divisble by 7, and thus, so is 364!
Another example 1792 2 is the last digit multiply it by 2and subtract it from original number

1792
-4
1 7 5 repeat once again 5x2=10
- 10
_ 7 7 is divisible by 7 so is 1792

Division by 8
If the number's last 3 digits are 000 or if they form a 3-digit number evenly divisible by 8, then the number itself is divisible by 8. How about 56,789,000,000? Last 3 digits are 000, so it's divisible by 8. Try 786,565,120. The last 3 digits, 120, divide by 8 into 15, so yes, the whole number is divisible by 8.

Division by 9
Sum the number's digits. If it divides by 9, you're in luck. As with the tests for 3 and 6, you can keep adding numbers until you're left with only one digit. 9873 , 9+8+7+3=27 =2+7 =9 9873 is divisible by 9

Division by 10
Any number that ends in 0 is evenly divisible by 10.

Division by 11
Here are four ways for different types of numbers:

1. If the sum of every other digit, starting with the first, is equal to the sum of every other digit starting with the second, then the number is evenly divisible by 11. Try 13057. 1+0+7 = 3+5, therefore it should divide evenly by 11. And indeed it does: 13057 / 11 = 1187. 1+0+7-3-5=0
2. If the digits are different, count them from the right and then add the numbers in the odd positions and the even positions. Subtract the smaller number from the larger. If the difference is evenly divisible by 11, so is your original number. Take the number 181,907. The numbers 8,9, and 7 are in the odd positions. They sum to 24. The numbers 1,1, and 0 are in the even positions. They sum to 2. Subtract 2 from 24 to get 22. 22 divides by 11 into 2, so 181,907 is evenly divisible by 11.

Division by 12
If the number can be evenly divided by 3 and 4, the same can also be said for 12. Use the methods for Division by 3 and Division by 4 above. If they both work, your number is also evenly divisible by 12.

Division by 13
Multiply the last digit by 4 and add it from remaining digits Is this number evenly divisible by 13? If it is, then your original number is evenly divisible by 13. Try 598

598
+32 (8x4=32)
9 1 Repeat once again
+ 4
13 so it is divisible by 13

Division by 15
If the number can be evenly divided by 3 and 5, the same can also be said for 15. Use the methods for Division by 3 and Division by 5 above. If they both work, your number is also evenly divisible by 15.

Division by 17

Multiply the last digit by 5. Subtract this answer from the remaining digits. Is this number evenly divisible by 17? If it is, then your original number is evenly divisible by 17. Try 663

663
- 15
51 it is divisible by 17 so 663 is divisible by 17

Division by 19

Multiply the last digit by 2. Add this answer from the remaining digits. Is this number evenly divisible by 19? If it is, then your original number is evenly divisible by 19. Try 741

741
+ 2
76
+ 12
19 it is divisible by 19 so 741 is divisible by 19

Division by 23

Multiply the last digit by 7. Add this answer from the remaining digits. Is this number evenly divisible by 23? If it is, then your original number is evenly divisible by 23. Try 667

667
+ 47
113

+ 21
23 it is divisible by 23 so 667 is divisible by 23

Division by 24
If the number can be evenly divided by 3 and 8, the same can also be said for 24. Use the methods for Division by 3 and Division by 8 above. If they both work, your number is also evenly divisible by 24.

Division by 29

Multiply the last digit by 3. Add this answer from the remaining digits. Is this number evenly divisible by 29? If it is, then your original number is evenly divisible by 19. Try 667

667
+ 21
87 _
+ 21
29 it is divisible by 29 so 667 is divisible by 29

Division by 31

Multiply the last digit by 3. Subtract this answer from the remaining digits. Is this number evenly divisible by 31? If it is, then your original number is evenly divisible by 31. Try 744

744
- 1 2
62 _
6
0 it is divisible by 31 so 744 is divisible by 31

Division by 33
If the number can be evenly divided by 3 and 11, the same can also be said for 33. Use the methods for Division by 3 and Division by 11 above. If they both work, your number is also evenly divisible by 33.

Division by 36
If the number can be evenly divided by 4 and 9, the same can also be said for 36. Use the methods for Division by 4 and Division by 9 above. If they both work, your number is also evenly divisible by 36.

t

Division by 37

Multiply the last digit by 11. Subtract this answer from the remaining digits. Is this number evenly divisible by 37? If it is, then your original number is evenly divisible by 37. Try 925

925
- 55
37 it is divisible by 31 so 925 is divisible by 37

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Lesson12

Division by 11

Here we learn division by 11 It is very easy and you can do it mentally very fast

First we check particular number is divisible by 11 or not We already discuss this method in lesson 11 .There are only two possibilities either that number is divisible by 11 or it is not divisible by 11

Number is divisible by 11 Let the number is 47894

<u>4</u> 7 <u>8</u> 9 <u>4</u>

total of odd position -total of even position

(4+8+4)-(7+9)

16 -16 =0 so this number is divisible by 11

Steps Write number 47894 ,take last digit ie 4 Write it extreme right ....4. Now subtract 4 from second last digit ie9-4=5 write before 4 our partial answer is ....54 .Again subtract 5from8 8-5=3 Write before ......54 .Now our partial answer is ....354 ,Repeat these steps up to first digit and get required answer .Here we subtract 3 from7 , 7-3=4 Our partial answer is 4354 Again subtract 4 from4 4-4=0 0 is indication we are on right track So 4354 is required answer

4	7	8	9	4	
4-4	7-3	8-5	9-4		
0	4	3	5	4	start from here

=04354

Let us see one more example Here we also consider carry

$\frac{5225}{11}=$

5	2	2	5	
4-4	11-7	12-5	|	
|	|	|	|.	
0	4	7	5	start from here

= 0475 is our required answer

Number is not divisible by 11 Same method as we discuss above but we make first number divisible by 11. Let us take a number 7539 This number is not divisible by 11 because difference of total of odd and even position is not 0 or 11 .22.....

<u>7</u> 5 <u>3</u> 9 (7+3)-(5+9) = 10 -14 =--4

If we subtract 4 from this number ,now this number is certainly divisible by 11 . And that number is 7539-4 =7535 By this way we find remainder .Here our remainder is 4

$$\frac{7539}{11} = \frac{7535}{11} + \frac{4}{11}$$

Now we can solve first part as we done above and for second part it come after decimal .For it just subtract 1 from remainder here it is 4 so 4-1=3 and for second place subtract 3 from 9 ,9-3=6 ie our answer now become . 36 This .36 repeat again and again ie .36363636......

7	5	3	5
6-6	14-8	13-5	
0	6	8	5

0685.363636...... it is our required answer

See one more example 7895
Difference of odd position and even position

7+9-8-5 =16 -13 =3 If we add 3 to 7895 it become fully divisible by 11 but our number is increased by 3 so we can not add 3 Here we adopt another method instead of adding we subtract 3 from 11 and proceed as above here our remainder is 11-3 =8
So we subtract 8 from 7895

$$\frac{7895}{11} = \frac{7887}{11} + \frac{8}{11}$$

$$717 + \frac{8}{11}$$

Here our remainder is 8 subtract 1 from it 8-1=7 and this 7 is subtract from 9 9-7=2 so after decimal72 .... ie .727272...

Required answer is **717.7272727....**

Be careful in addition and subtraction of remainders

-

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Lesson13

Division by 9

Division by 9 is also very easy and you can do it very fast mentally

- **24 / 9 = 2 remainder 6**

  The first figure of 23 is 2, and this is the answer.
  The remainder is just 2 and 3 added up!
- **42 / 9 = 4 remainder 6**

  The first figure 4 is the answer
  and 4 + 3 = 7 is the remainder **-** could it be easier?

- **133 / 9 = 14 remainder** 7

  The answer consists of **1,4** and **7**.
  1 is just the first figure of 133.
  4 is the total of the first two figures 1+ 3 = 4,
  and **7** is the total of all three figures 1+ 3 + 3 = 7.

  See one more example

  1232 / 9 = 136 remainder 8

- **842 / 9 = $8_1$2 remainder 14 = 92 remainder 14**

  Actually a remainder of 9 or more is not usually
  permitted because we are trying to find how
  many 9's there are in 842.

  Since the remainder, 14 has one more 9 with 5
  left over the final answer will be **93 remainder 5**

  See one more example

- **123456 / 9 = $136_1 16_1$ remainder 3 = 13717 remainder 3**

Answer in decimal

Take above example **123456 / 9 = 13717 remainder 3**

It is very very easy put decimal after 13717 write remainder 3 and repeat it again and again .

13717 .3333333333333.. It is our required answer .

With the help of this concept we can solve some other problems
Take few examples
1234/81 = ???? It is just 1234 we divide 1234 by 9 two times and get required answer

-

9x9

= $\frac{136_1 \text{ remainder } 1}{9}$ = $\frac{137.1111111}{9}$ =1 4_1.23456 =15.23456

362/63 =?????? $\frac{362}{9x7}$ = $\frac{3_1\ 0 \text{ remainder } 2}{7}$ = $\frac{4\ 0.222222}{7}$ = 5.7460

489/45 = ????? Apply little presence of mind here ,Multiply 489 and 45 to 2

$\frac{489x2}{45x2}$ = $\frac{978}{90}$ = $\frac{108\text{remainder } 6}{90}$ = $\frac{108.66666}{10}$ = 10.866666

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Lesson14

Division by a Number Ending with 9

This concept is a real gem of Vedic Maths .After learning this concept you experience lot of thrill in yourself .It is very easy and you can find it very useful for your calculations

Denominator ending with 9

Let our question is divide 259 by 19 up to 6 places of decimals

First we solve it by conventional method

```
     13.631578
19)  259
     19
      69
      57
      120
      114
        60
        57
         30
         19
          110
           95
           150
           133
             170
              152
                18
```

I think all are well conversant in this method, so no need to explain further .
Now we learn the division of 19 by Vedic method

$\frac{259}{19}$=????

Steps $\frac{259}{19} = \frac{25.59}{2}$ Put proper decimals and make 19 to20 ,take next number ie 19 to 20 ,29 to30 , 39 to 40 119 to 120 etc etc

Now it is much easier to divide by 2 in comparision of 19 or 3 to 29 or 4to 39
Now we learn steps

1
2 59 First we divide 2 by 2 1 is Quotient no Remainder

13
2 **5**9 Now add 1 to 5 ,6 is our gross so divide 6 by 2, 3 is Quotient, no Remainder

13 6
2 **5**9 Now add 3 to 9 , ie 12 is now our gross so divide 12 by 2 6is Quotient and no remainder

13 63
2 **5**9 Now nothing to remain in our main number ie259 so take 6 directl
now 6 become our gross so divide 6 by 2 3is Quotient No remainder

13 631
2 59 Take 3 directly , now 3 become our gross so divide 3 by 2 1 is Quotient
1 1 remainder

13 6315
2 59 Take Quotient 1 put it before remainder 1 ,it looks like 11 ,now 11
11 become our gross so divide 11 by 2 5 is Quotient 1 remainder

13 63157
2 59 Take Quotient 5 put it before remainder 1 ,it looks like 15 ,now 15
15 become our gross so divide 15 by 2 7is Quotient 1 remainder

13 631578
2 59 Take Quotient 7 put it before remainder 1 ,it looks like 17 ,now 17
1 become our gross so divide 17 by 2 8is Quotient no remainder

Repeat the above steps if you want to find the values further.

Now our answer after proper decimal is 13.631578

Now check both answers whether two answers are same ??????

Let us see one more example $\frac{392}{29}$ =? Make denominator 29 to 30

$\frac{392}{30}$

1 3 5 1 724
= 3 9 2 /3
10 ,15 ,5,21,7,12

= 13.51724 Answer

See one more example $\frac{485}{39}$

$\frac{485}{40} = \frac{48.5}{4}$

1 2 4 3 5
48 5 /4
9 17 14 23 53

= 12.435 Answer

Now see one example of carry $\frac{297}{29} = \frac{297}{30} = \frac{29.7}{3}$

9 12 4 1 3 =102413
2 9 7 /3
2 when we add 16 to 2 consider it 20
16 not 2 , it is why 2(20)+16 =36. Eliminate
36 12 4 11 1of 12 consider 1 as carry ,1
add to 9 , become 10

Now answer is 10.2413

Let us see one example of 3 digit $\frac{3469}{129} = \frac{3469}{130} = \frac{346.9}{13}$

2 6 8 9 1
346 9 /13
88 10 consider its 100 not 10
15
115 118 19

= 26.891 Answer

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Lesson15

Division by Number Ending 3 & 7 or Any Other number

I think all of you people have become well conversant in division of 19 as we learned it in previous lesson .Now with the help of division of 19 we learn division by 13 and 17

Division by ...3

Let see one example

456 =????
13 Multiply nume./ demo. each by 3

456 x3 = 1368 = 1368 =136.8
13x3 39 40 4

~~3 41 0 7 6~~ = 35076
13 6 8 /4
1 9 3
12
42 2
10
30 27

Our required answer is 35.076

Let see one more example 893 = 893x3 =2679 = 2679 = 267.9
23 23x3 69 70 7

-

3 8 8 2 6
267 9 /7
5
10
60 4
17
57 18 42

Our required answer is 38.826

-

Division by ...7

Learn with the help of example

268 = ???? Multiply nume. /demo. each by7
27

268 x7 = 1876 = 1876
27x7 189 19

9 9 2 59 2
1876 /19
16 15 _ 175 ,49 ,112 ,175,49 Now we write whole rough

work in single line for fast speed

9.92892 is our required answer

Division by Any number

$\frac{268}{43}$ =?? $\frac{268 \times 3}{43 \times 3} = \frac{804}{129} = \frac{804}{13}$

6232
804 /13
2 1 0 _30,42,33

Answer =6.232

One more example

$\frac{398}{23} = \frac{398x3}{23x3} = \frac{1194}{69} = \frac{1194}{7}$

17304
1194 /7
4 1 0_50 ,1 1 1_21,30

Answer =17.304

Division by this method is not always feasible .Some time it creates problem also

$\frac{298\ 8}{221} = \frac{2988x9}{221x9} = \frac{26892}{1989} = \frac{26892}{199}$

Division by 211or division by 199 has any difference , even division by199 takes your more time .So use this method carefully . It gives wonderful results with presence of mind

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Lesson16

Subtraction & Addition

Subtraction is often faster in two steps instead of one.

Example: 427-38=(427-27)-(38-27)=400-11=389

GOLDEN RULE "First remove what's easy, next whatever remains".
Example : 2931 -1243 =2800-1200+131 -(31 +12)
=1600 +100-12 = 1600 +88 =1688

Example: 4878 - 496 = 4878+4 -(496 +4)
4882 -500 = 4382

Example: 1048-189=1048+11 -(189+11.)
= 1059 -200 =859

Example: 1893-1245 =1893+7-(1245+7)
1900 -1252 =648

Addition.

Addition is often faster in two steps instead of one.

Example: 495+38=(495+5)(38-5)=500+33=5

GOLDEN RULE "First add what's easy, next whatever remains

Example: 1256 +289 = 1300 +289-44
=1300+245=1545

Example: 1893+1245 =3193+7+(45-7)
3200 +38=3238

Sometime it is easy and quite fast if we start with higher digit
Example: 952 +687 =1500+52+87
=1600+52-13
=1600+39
=1639

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Lesson17

Addition & Subtraction of Fractions

Use VERTICALLY AND CROSSWISE to write the answer straight down!

$$\frac{2}{3} + \frac{8}{4} =$$

$$\frac{2x4 + 8x3}{12} = \frac{8+24}{12} = \frac{32}{\cancel{12}} = \frac{16}{6}$$

Multiply crosswise and add to get the top of the answer:
2 x 4 = 8 and 8 x 3 = 24. Then 8+ 24= 32.
The bottom of the fraction is just 3 x 4 = 12
You multiply the bottom number together

$$\frac{2}{7} + \frac{7}{9} = \frac{18 + 49}{9 x7} = \frac{67}{63}$$

Subtracting is just as easy: multiply crosswise as before, but subtrac

3/ 5 -2/ 6

$$\frac{18-10}{30} = \frac{\cancel{8}}{\cancel{30}} = \frac{4}{15}$$

Now see the addition of these compound fractions

$$1\frac{1}{2} + 2\frac{1}{3} =$$

Here 1 and 2 are whole numbers and 1/2 and 1/3 are fractions .Addition of these compound fractions are quite easy .Add whole numbers and fractions separately then write both together

$$1+2 = 3 \quad . \quad \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6 \quad 6} = \frac{5}{6}$$

$$= 3\frac{5}{6}$$

It is our required answer

In subtraction do as before but subtract instead of addition

$$5\frac{2}{3} - 2\frac{2}{4} =$$

$$5- 2 =3 \qquad \frac{2}{3} - \frac{2}{4} = \frac{8 - 6}{12} = \frac{2}{12} = \frac{1}{6}$$

$=3\frac{1}{6}$

It is our required answer

Square of Fraction

To square any compound fraction containing 1/2, like 41/2 for instance
Multiply the whole number by the next higher whole
and append 1/4 to the product.
Thus, 4 1/2 x 4 1/2 = 20 1/4. (4 x (4+1) = 20; tag on 1/4 to get 20 1/4.)

Multiplication of fraction

To multiply any two like numbers with fractions that sum to 1
6 3/4 x 6 1/4), multiply the whole number by the next highest number (6 x 7)
and append the product of the fractions (3/4 x 1/4).
In the case of 6 3/4 x 6 1/4, 6 x 7 = 42
Then append the product of 3/4 x 1/4, 3/16. Thus, 42 3/16.

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Lesson18

Percentage

You can find percentage very fast, no calculations, write only required answer directly

10 % :	Just divide the number by 10	Find 10% of 229 = 229/10 =22.9
12.5%:	Just divide the number by 8	Find 12.5% of 208 = 208/8 =26
$33\frac{1}{3}\%$:	Just divide the number by 3	Find $33\frac{1}{3}\%$ of 270 = 270 /3 =90
25%	Just divide the number by 4	Find 25% of 296 = 296/4 =74
50%	Just divide the number by 2	Find 50% of 296 =296/2 148
75%:	Just divide the number by 3/4	Find 75% of 208 = 208x3/4 =624/4=156
37.5% :	Just divide the number by 3/8	Find 37.5% of 240 = 240x3 /8 =90
15%	Just multiply by 3 and divide by 20 and put proper decimal	Find 15% of 296 = 296x3/20 =888/20=44.4
45 %	Just multiply by 9 and divide by 20 and put proper decimal	Find 45% of 122 = 122x9/2 = 1098 =54.9 Hint use multiplication by 9 Lesson 5
55%	Just multiply by 11 and divide by 20 and put proper decimal	Find 55 % of 132 = 232 x11/20 = 2553/20=127.65 Hint use multiplication by 11 Lesson 2(By using same method we can calculate %of 35 ,85 ,95)
10.25%	Calculate in two parts 10% & .25%	Find 10.25% of 240 : 10% of 240 + .25 % of 240 = 240/10 +240/400 = 25 +.6 = 24.60
25.25.%	Calculate in two parts 25 % & .25%	Find 25.25%of 280 : 25%of 280 + .25%of 280 = 280/4 + 280/400 = 70 + .7 = 70.7
56%	Calculate in two parts 55%&1%	Find 56% of 780 55%of 780 +1 %of 780 = 780x11/20 +780/100 = 39x11 +7.8 = 429 +7.8 = 436.7
78%	calculate in two parts 80%-2%	Find 78% of 1640 80 %of 1640 -2%of 1640 = 1640x8/10 -1640x2/100 = 1312 -32.8 = 1279.2
12%	Calculate in two parts 10 %+2%	Find 12 %of 268 26.8 +5.36 =32.16
26%	Calculate in two parts 25%+1%	Find 26 %of 292 292/4 + 2.92 =73+2.92 =75.92
89 %	Calculate in two parts 100%-11%	Find 89 % of 256 256 - 11x256/100 =256 -28.16 =227.84

Apply little presence of mind and calculate percentage as fast as you can.

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Lesson19

Under Root

Now learn a very easy method -how to find under root of any number
Let see this example; Find the under root of 39

Here we know that perfect square near to 39 is 36 and under root of 36 is 6
So under root of 39 is more than 6 Let we take under root of 39 is 6.2 by approximation
Divide 39 by 6.2

$$\frac{39}{6.2} = 6.290$$

Now take average of 6.290 and 6.2 and again divide 39 by this average
(6.290+6.3)/2= 6.295

$$\frac{39}{6.295} = 6.195$$

Now this 6.195 is near to under root of 39 again we take average of this number with last 6.290
6.290+6.195/ 2 =6.242 divide 39 by this 6.242

$$\frac{39}{6.242} =6.247$$

This 6.247 is very very near to our required answer
If we again repeat this step one more time our answer is more accurate

See one more example : Find the under root of 89

$$\frac{89}{9.4} = 9.46$$

Here we take 9.4 by approximation considering 9 is under root of 81

$$\frac{89}{9.43} =9.43$$

9.43 is average of 9.46 and 9.4 . 9.43 is under root of 89 up to two places of decimal

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Lesson20

Miscellaneous Multiplication

Here I tell you some miscellaneous methods of multiplication from 18 to 101
These methods are based on presence of mind. As you know multiplication/division of 2, 5 ,9,10,11, 25,50, is very easy in comparison of other numbers . So we use these multiplications /divisions in these methods

18 x296 =(20-2)x296 =5920-592=5328

19x296 = (20-1)x296 =5920-296=5328

21x296 = (20+1)x296 =5920+296=6216

22 x296 =(20+2)x296 =5920+592=6512 or 11x2x296

23x296 = (2x22+1)x296 =6212+296=6808 or 29600/4-2x296 =7400-592=6808

24x296 = (100/4-1)x296 =7400-296=7104

25x296 = (100/4)x296 =7400

26x296 = (100/4+1)x296 =7400+296=7696

27x296 = (30-3)x296 =8880-888=7992

28x296 = (30-2)x296 =8880-592=8288

29x296 = (30-1)x296 =8880-296=8584

31x804 = (30+1)x804 =24120+804=24924

32x804 = (30+2)x804 =24120+1604=25728

33x804 = (30+3)x804 =24120+2412=26532 or 3x11x804

34x804 =(3x11+1)x804 =27336

35x804 =(25+10)x804 =(100/4 +10)x804 or 70/2 x804

36x804 =(3x11+3)x804 or (4x10-4)x804

37 x..... =(40-3) x......

38x..... =(40-2)x.......

39x....=(40-1)x.....

41x....=(40+1)x....

42x.... (40+2)x....

43x...=(40+3)x....

44x....=(4x11)x....or (40 +4)

45x..... = 90/2

46x.... =(90/2+1)x...

47x..... =(90/2+2)x...

48x..... =(100/2-2)x....

49x.....=(100/2-1)x....

50x.....=100/2x....

51x.....=(100/2+1)x.....

52x... =(100/2+2)x

53x....= (110/2 -2)x

54x....= (110/2+1)x

55x .. = (110/ 2)x...

56 x.. = (110./2+1)x...

57 x.. = (60-3)x..or(2x3x10-3)

58x....= (60-2)x....

59x... = (60-1)x...

61x.. = (60+1)x...

62x... = (60+2)x

63x = (70-7)x.....

64x... = (130/2-1)x...

65x = (10+11x10/2)x...

66x = (2x3x11)x

67x = (2x3x11+1)x

68x = (70-2)x or (101 -3x11) Multiply of 101 is like 100 if there is only two digit repeat two time same number
36x101 =3636 if three digit 333x101= 33 (3+3)33=33633

69x = (70-1)x

71x =(70+1)x

72x = (80-8)x or(300/4 -3)x or (6x11+6)x

73x = (300/4-2)x

74x = (300/4-1)x

75x = 300/4

76x =(300/4+1)x

77x =7x11x

78x =(80-2) x or(100-2x11)x

79x =(80-1)x or (101 -2x11)

81 x = (80+1)x or 9x9

82x =(80+2)x

83x =(80+3)x

84x =(77+7)x

85x = (300/4+10)x

86x = (8x11-2)x

87x = (8x11-1)x

88x= 8x11x

89x=(90-1)x or (100-11)x

90x =9x10x or(100-10)x

91x = (101-10)x or (90+1)x

92x =(90+2)x or (100-8)x

93x= (90+3)x or (100-7)

94x=(100-6)x

95x=(100 -10/2)x

96x=(100-4)x

97x=(100-3)x

98x=(100-2)x

99x=(100-1)x

101=(100+1)x

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